Collective Argumentation and Modelling Mathematics Practices Outside the Classroom

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An important aspect of bringing about change in the mathematics classroom is gauging the efficacy of the change in bringing about learning that has application outside the school classroom. The research reported in this paper is situated within an on-going study where over 20 teachers of mathematics in the middle years of schooling are using the practices of Collective Argumentation to bring about change in their classrooms. This paper reports on one aspect of the study that sought to explore whether students who use Collective Argumentation on a regular basis in their classrooms view mathematics as providing a forum where personal understandings can be expressed, re-considered, shared and co-authored when they go about knowing and doing mathematics in a novel context – an inter-school mathematics modelling challenge. The results of the exploration are discussed and situated within the context of the findings of the on-going study.

The Mathematics Teacher's Lament

"I spent 3 weeks teaching this and the students do not have a clue what I am talking about."

In the day-to-day operation of a school, comments such as the one above are common. When it comes to mathematics, there is evidence to support the idea that many students are disinterested and unwilling to engage in the teaching and learning process (Boaler & Greeno, 2000). Often students question what is being taught in class because they do not see the relevance of what they are doing (Pajares & Graham, 1999). Students question, "Why do we have to know this?"; "Where am I ever going to use this?" Yet it is interesting that many mathematics teachers persist with teaching and learning practices that perpetuate the view that mathematics understandings are transmitted rather than constructed (Schoenfeld, 2004).

In terms of teaching for understanding, Perkins (1992) has identified shortfalls in education. One shortfall that he identifies refers to "inert knowledge" (p. 22), that is, knowledge that is only able to be articulated by the student if the right stimulus is provided by the teacher. A question is asked and that triggers a response that allows the student to give the correct answer. At this level of knowledge acquisition, the teacher may incorrectly assume that a student has developed understanding of a concept only to find later that the student is unable to apply the knowledge to a novel situation.

Another shortfall that Perkins (1992) has identified is "ritual knowledge" (p. 25). In displaying this type of knowledge, students have learnt to play the school game. They are able to use the language of mathematics and they are able to use the correct mathematical procedures to manipulate mathematical expressions such as equations, but they have difficulty modelling the mathematics, for example, building equations, when they are embedded in a novel context.

If, as Perkins (1992) argues "learning is a consequence of good thinking" (p. 8), suitable problem solving routines need to be built in the classroom that allow students to develop understanding through the use of good thinking skills. Students need to be encouraged to use thinking routines (Richart, 2002, p. 89), that may become part of their repertoires of practice when coming to know and do mathematics. The routine needs to be simple, explicit, and provide students with a mechanism to engage with the task, construct meaning, build a solution, and communicate that solution to others (Richart, 2002, p. 90). One such routine that has been taken up by teachers to privilege student understanding in the mathematics classroom is Collective Argumentation (Brown & Renshaw, 2000).

Collective Argumentation

Collective Argumentation involves the teacher and students in ways of coming to know, do, and value mathematics that reflect the investigative processes and ways of interacting employed by the mathematical community. In simple terms, collective argumentation involves the teacher and students in small group work (two to five students per group) where students are required, initially, to "represent" a problem individually by using pictures, diagrams, drawings, graphs, algorithms, numbers, etc. Students are then required to "compare" their representations with those of other group members. This phase of individual representation and comparison provides the potential for differences in understanding of curriculum content to be exposed and examined. Subsequent talk by the students regarding the appraisal and systematisation of representations is guided by the keywords – "explain", "justify", "agree". Finally, moving from the small group to the classroom collective, the thinking within each group is validated for its consistency and appropriateness as it is presented to the whole class for discussion and validation.

The Teacher's Role in Collective Argumentation

The teacher has an active role throughout each phase of Collective Argumentation. The tasks of the teacher include: (a) allocating management of the problem-solving process to the group; (b) facilitating peer co-operation by reminding students of the norms of participation; (c) participating in the development of conjectures and refutations; (d) modelling particular ways of constructing arguments; (e) facilitating class participation in the discussion of the strengths and weaknesses of a group's co-constructed argument, (f) introducing and modelling appropriate mathematical language; and (g) providing strategies for dealing with the interpersonal issues that may arise when working with others.

This paper explores the effects of Collective Argumentation in making visible students' understandings as they go about knowing and doing mathematics in a novel context – an inter-school mathematics modelling challenge. Specifically, the paper seeks to explore whether a group of students from a Collective Argumentation classroom see mathematics as providing a forum where personal understanding is privileged, that is, as providing a space where personal understandings can be expressed, re-considered, shared and co-authored.

Method

This paper arises from an on-going study of teachers' appropriation of the practices of Collective Argumentation into their everyday teaching of mathematics and/or science. The study is being conducted over a three-year time frame with 20 elementary and middle school teachers of mathematics and/or science from six schools located in South-East Queensland. The study employs a sociocultural design, based on a "design-experiment" (see Schoenfeld, 2006). The "design-experiment" is an extension of Vygotsky's (1987) experimental-developmental method that was designed to capture the determining influence of social and cultural processes on learning and development. From this perspective, the activity of the students, the activity of the teacher/researcher, and the co-constructed activity of the classroom, interrelate at a number of levels to create the "life context" of the mathematics classroom. A "design-experiment", therefore, requires multiple sources of data to be collected and involves prolonged, systematic inquiry into change through engagement in collaborative cycles of analysis, design, implementation, assessment, and reflection. The authoring of this paper is an artifact of this cyclical process.

Participants. The teacher, whose students are the focus of this paper, had been using the practices of Collective Argumentation to inform his teaching of mathematics to students for one school year. The teacher taught at a P-12 school located in a middle-class suburb of a major city. Thirteen students (seven girls and six boys) from this teacher's Year 5/6 class and 14 students from other schools had been encouraged by their teachers to participate in the Year 6/7 section of an inter-school mathematics modelling challenge. Three of these students (two girls and one boy) – Helen, Nicole, and Neil – form the focus of this paper. All three students were high-achievers in mathematics.

The Challenge. The challenge was conducted over a 2-day period at the campus of a local university during the last week of November and was attended by 220, Year 4 to Year 11 students from South-East Queensland. Each day of the challenge lasted from 9:00am till 3:00pm and consisted of three sessions broken by morning tea and lunch. During the challenge, students were allocated to a group of four students and invited to work with mathematics educators on authentic mathematical modelling tasks appropriate for their year level. At the completion of the challenge each member of the group, at each year level, judged to have provided the best mathematical model of a solution to a task was awarded a plaque and a calculator valued between $50 and $150.

The Task. The students who form the focus of this paper were allocated to the same group. The group comprised Helen, Nicole, Neil and Aaron (a student from a local state school). Over the two days of the challenge, the students were engaged with the task of designing, building to scale, and mathematically modelling a mini-golf course. Each group was required to design a mini-golf hole on graph paper – complete with blockers, tunnels and other obstacles – and create a theoretical hole-in-one path of the ball such that each angle of incidence equalled the corresponding angle of reflection. Each group was required also to represent their mini-golf hole design on graph paper, provide a spreadsheet showing the segment angles, slopes, and linear equations, and provide a short journal entry of their experience with the challenge. Each group received a poster board, a piece of green felt, wooden blocks, cardboard tubes, and a marble along with graph paper and a criteria checklist. Four computers, connected to the internet were available for the students to use. Clarification of task requirements was provided to each group by a mathematics educator, however no direct teaching of task content was provided.

Data Collection. The targeted group was video- and audio-taped by research assistants at three pre-determined one-hour segments of the mathematics challenge. The first recording occurred in the middle-session of the first day of the challenge, recording sessions two and three occurred in the morning and middle-sessions of the second day. However, the research assistants were present for the entirety of the challenge and video-recorded the targeted group outside pre-determined times when they thought that something of interest to the research was happening. At the conclusion of the challenge, all video- and audio-tapes were transcribed and names were replaced with pseudonyms. Consent was sought and gained from the participants for the transcripts to be used for research purposes.

The sections of transcript provided in this paper were taken from the second pre-determined taping session and from a moment of interest in the challenge when students communicated to other students outside their group. These segments of text were chosen for analysis because they provide instances of students talking about what they learnt and accounts of how they came to know the conceptual elements of the task.

Data Analysis. Bakhtin's (1986) notion of "voice" was used to analyse the transcripts. Bakhtin (1986) formulated a theory of voice that emphasized the active, situated, and functional nature of speech as it is employed by various communities within a particular society. Taking the notion of "utterance" rather than "word meaning" as a basic unit of communication, Bakhtin maintained that in dialogue with others, people align themselves within different speaking positions or voice types as they produce or respond to an utterance or a chain of utterances. Such voice types reflect the social ways of communicating that characterize various group behaviors (eg., professional communities, age groups, and socio-political authorities) that a person has had the opportunity and/or willingness to access. As such, "voice" as used in this paper, encompasses "what" is being said, the "way" in which it is spoken, and the positioning of speakers in relation to the authority framework established within the communication.

Analysis and Discussion

We enter the mathematics modelling challenge when Helen, Nicole, Neil and Aaron (a student from a different school) are preparing a short journal entry of their experience with the challenge. The extract is taken from the second targeted data collection session held in the morning of the second day of the challenge (see Table 1).

Table 1
Maybe our Whole Group Learnt About it

Turn	Speaker	Text
01	Nicole	Aaron learnt, what did you learn?
02	Aaron	I learnt lots.
03	Nicole	Well then tell us.
04	Aaron	I learnt about slope.
05	Neil	Maybe our whole group learnt about it.
06	Helen	I didn't (learn about slope), I had to do it ($y = mx + 3$).
07	Neil	(I learnt) About the equations.
08	Helen	$y = mx + 3$

We enter the script where Nicole is recognising Aaron's "belonging" in the group by asking him what he had learnt from engaging with the mini-golf task. Instead of accepting Aaron's general response (turn 2 - *I learnt lots*) and then moving to record the responses of the other members of the group who were from her school, Nicole encourages Aaron to be reflective and to consider the specifics of what he had learnt (turn 3 – *Well then tell us*). This action reflects a reason Nicole's teacher gave for taking up Collective Argumentation in the classroom, “We want to encourage our students to be reflective and consider how the various concepts (in mathematics) are related”.

Collective Argumentation privileges this level of understanding by requiring students to explain and justify their learning on a regular basis, therefore, making knowledge public. Explaining and justifying involves the gathering and sharing of evidence that satisfies disciplinary constraints associated with coherence and logic. Explaining and justifying allows students to become conscious of others’ ideas and points of view, allowing processes of thought as well as products to become visible.

This privileging of reflecting on understanding continues as the other members of the group comment on their personal understandings relating to “slope”. Here we see students being reflective, considering what they have learnt (turn 7- *About the equations*) and what they did not learn (turn 6 – *I didn’t, I had to do it*). However students saying they have learnt it does not mean they understand it, as Helen reveals in the next sequence of text (see Table 2).

Table 2
I Knew Something that you Didn’t Know

Turn	Speaker	Text
09	Nicole	Helen, what did you learn today?
10	Helen	y = mx + 3
11	Nicole	Didn’t you already know that?
12	Helen	No, how to do it, like I knew what it (slope) was, I just didn’t know how to do it (slope).
13	Nicole	Didn’t you know how to do it (slope)?
14	Helen	You (Nicole) didn’t.
15	Nicole	Yes I did, well I knew how to do it the obvious way, I knew how to do it on a graph, but on quadrant things (quadrants of a full grid).
16	Neil	I knew something that you didn’t know.

Here we see Helen and Nicole linking what they know, considering a different strategy (using y = mx + 3 [turn 10] or graphing a line on a grid [turn 15]) and recognising they are doing the same thing. Through this text, we see Helen and Nicole transferring the mathematical tools they had leant in the classroom to this context and recognising that there are different ways of applying those tools and different levels of knowing about and using mathematical tools.

Collective Argumentation privileges the recognition of multiple representations of a mathematical idea through requiring students to represent a solution or idea about a task individually and to compare their representation with others. When students complete a brief written response to a text, or a solution to a problem, or an evaluation of the effectiveness of an experiment, they are more likely participate in any discussion that follows, ask questions of others, share ideas with others, and to self-monitor their understanding (Gaskins, Satlow, Hyson, Ostertag, & Six, 1994). Comparing representations allows students to see what is the same and what is different about their ideas and interpretations. In the process, it can help students learn by making them view concepts from different perspectives, and can be affirming as students see congruence between ideas and representations (Feltovich, Spiro, Coulson, & Feltovich, 1996).

Through recognising that Nicole is using a graphical approach to completing the task and that Helen is using an algebraic approach, the students pave the way for relating procedural to conceptual understanding, as illustrated in the following sequence (see Table 3).

Table 3
We Used it, but we Didn't Know How

Turn	Speaker	Text
17	Nicole	Neil, what did you learn?
18	Neil	I learnt that, I learnt just that (y = mx + 3).
19	Nicole	What do you mean just that? y = mx + 3?
20	Neil	Just write everybody learnt that (y = mx + 3), because we all did learn that, yeah everybody learnt it.
21	Nicole	I need an eraser.
22	Neil	So you don't have to write just Aaron (learnt y = mx + 3) cause we all learnt it.
23	Nicole	Did anyone else learn anything that's not there (in the journal entry)?
24	Neil	Um maybe we …
25	Nicole	How to use slope or anything?
26	Helen	That (slope) is part of the equation.
27	Neil	Yeah, that's part of the equation. Let's see, what about how to …
28	Aaron	Did you know that equation (y = mx + 3) before we came (to the challenge)?
29	Nicole	We used it (y = mx + 3), but we didn't know how.
30	Helen	That's how to find out 'm'.

Once again (turns 17 & 19) a member of the group, Neil, is asked by Nicole to explicate what he learnt from engaging in the mini-golf task. Neil's admission that he learnt about slope (turn 22 - *So you don't have to write just Aaron cause we all learnt it*) marks a moment in the conversation when this grouping of students from two different schools, have become a group who are willing to take ownership of their learning. In so doing, links are made between "*how to use slope"* (turn 25) and the algebraic equation – y = mx + 3 (turn 26 – *that's part of the equation*) and between the concept of "slope" and its algebraic representation "m" (turn 30).

Collective Argumentation privileges linking conceptual with procedural understanding and linking individual with collective understanding by requiring that the group attain consensus about a response to a task that they can present to the whole class – a response that each member of the group understands. Consensus based on understanding is the end product of a process of considering and critiquing. Students negotiating a common understanding of a representation or idea take learning from the co-operative to the collaborative plane of learning.

This willingness to collaborate in the sharing of understandings continues in the next sequence of text, which was recorded in a moment of interest when the group extended its boundaries to include members from other groups undertaking the challenge. We enter the script where Helen has just explained to her group how to find the slope "m" in the equation y = mx + 3 (see Table 4). During the explanation, students from other groups gather around to listen. The students included Gail (another student from Nicole, Helen, and Neil's school).

Table 4
Let me Explain How to do "m"

Turn	Speaker	Text
31	Nicole	Yeah, I got that, I got it (y = mx +3).
32	Neil	Yeah I do (understand).
33	Nicole	Because I didn't really get it (y = mx + 3) before, but I understand now.
34	Gail	So you (Helen) just explained how to do 'm'?
14	Neil	Let me explain how to do 'm'.
15	Gail	He (points to a boy in her group) needs to figure out also how to do 'b' (the Y Intercept).
16	Helen	mx + 3, equals 'b' equals Y intercept.
17	Gail	He (a boy in her group) doesn't get (understand) it.
18	Helen	Whatever Y intercept is, is 'b'.
19	Gail	He (a boy in her group) doesn't understand.
20	Helen	Y intercept is when the Y, where the point Y is. Well then you (Neil) can explain it then.

The interaction of students in the above text is interesting for students engaged in an inter-school mathematics challenge. The challenge relating to the mini-golf course can be won by one group only. Helen, in demonstrating her understanding of how to find the slope of a line between two points provides an explanation that is attended to by students not in her group. Not only does Helen share her understanding with Nicole (turn 31) and Neil (turn 32), but also she receives a request from Gail (a member of another group) to explain again how to find the slope, as a boy in Gail's group does not understand. Neil requests permission to provide the explanation (turn 14). However, Helen simply revoices the main point of her explanation (turn 16). Upon receiving a signal from Gail that this revoicing is insufficient (turn 19), Helen gives Neil permission to explain. Neil goes on to present an explanation to the gathered audience that results in a number of students from different groups working together to build a model that they can use to make predictions.

Collective Argumentation privileges a view of mathematics as being about engagement in communal practice by requiring each group to present their agreed approach to the class for discussion and validation. Such presentations of group work permit students to engage with the conceptual content of a lesson at their level, to employ their own prior experiences, preconceptions, and language, and to distribute the nature of their knowing across a group rather than in a fashion that focuses on any one individual.

Conclusion

This paper set out to explore the effects of Collective Argumentation in making visible students' understandings as they go about knowing and doing mathematics in a novel context – an inter-school mathematics modelling challenge. The nature of the learning displayed by Helen, Nicole, and Neil as they engaged with the mathematics challenge of designing a hole-in-one mini-golf course, suggests that these students view doing the mathematics as providing a forum where personal understandings can be expressed, re-considered, shared, and co-authored – an unusual stance for students engaged in what might be viewed as a mathematics competition.

The nature of collaboration constructed by Helen, Nicole, and Neil displayed many of the characteristics of Collective Argumentation – a way of teaching and learning mathematics frequently employed by their classroom teacher. The above analysis of student-student interaction suggests that within this group's way of doing the mathematics challenge, understanding emerged around shared practice; that is, a collaborative space emerged where a voice of inquiry was enacted that privileged: (a) the relating of conceptual understanding to procedural understanding (e.g., determining slope and the Y intercept to build the equation, $y = mx + b$); (b) group ownership of learning over individual performance (e.g., designating new learning about slope to the whole group rather than just to an individual within the group); and (c) mathematising, that is, knowing not only the mathematics, but also how and when to use the mathematics, over "ritual" knowing (e.g., as suggested by Nicole's statement – *We used it ($y = mx + 3$) but we didn't know how*).

Within this collaborative space, on-going processes for adding meaning to the mini-golf task such as representing, comparing, and explaining were used by the students in a fashion that allowed their individual representations, ideas, and points of view to become products of the moment, able to be used by others to progress understanding. Students' interactions, as portrayed in the above transcripts, imply that within the collaborative space constructed by the students within the constraints of the mathematics modelling challenge, students not only co-constructed knowledge, but also developed an awareness of the "self" as operating with tools of mathematics (e.g., $y = mx + b$), of the self operating as a mathematician.

This paper has provided some evidence that students who experience Collective Argumentation on a regular basis in their classrooms do see mathematics as providing a forum where personal understandings can be expressed, re-considered, shared, and co-authored when they go about knowing and doing mathematics in a novel context. The interactions between Helen, Nicole, and Neil occurred within a real novel context centred around real mathematics challenges. Rather than displaying individual personalities engaged in competitive intellectual practice, Helen, Nicole, and Neil were drawn into a culture of inquiry that displayed distinct co-operative and collaborative relationships. However, this culture of inquiry did not happen by chance but is, we argue, a result of regular participation in the collaborative partnerships and relationships of Collective Argumentation.

In terms of the larger study in which these students and their teacher are situated, over 80% of the 20+ teachers who commenced doing Collective Argumentation in their classrooms in 2006, have carried these practices over into 2007. The major reasons provided by teachers who ceased participating in the study related to change of school, year level, or status within the school system. The teachers who have continued with Collective Argumentation in 2007 report an increased desire by their students to learn mathematics in the middle school years when doing Collective Argumentation and a corresponding decline in student behaviours that disrupt teaching-learning relationships. Teachers also report a growing need for professional development in the content domains of mathematics as they move away from using textbooks and structured mathematics lessons towards using the practices of Collective Argumentation to scaffold the teaching-learning relationship. In 2007, these teachers will be joined by eight more teachers from their respective schools who, after seeing these teachers successfully negotiate two rounds of reporting to parents, have expressed a desire for their students to use Collective Argumentation to come to know and do mathematics.

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